The 9th Dedekind Number

A case for FPGA Supercomputing

"The Nth Dedekind number is the number of Monotone Boolean Functions in N variables"

$$O(2^{2^n})$$

D(0) = 2	Dedekind (1897)
D(1) = 3	Dedekind (1897)
D(2) = 6	Dedekind (1897)
D(3) = 20	Dedekind (1897)
D(4) = 168	Dedekind (1897)
D(5) = 7581	Church (1940)
D(6) = 7828354	Ward (1946)
D(7) = 2414682040998	Church (1965)
D(8) = 56130437228687557907788	Wiedemann (1991)
D(9) = ?	

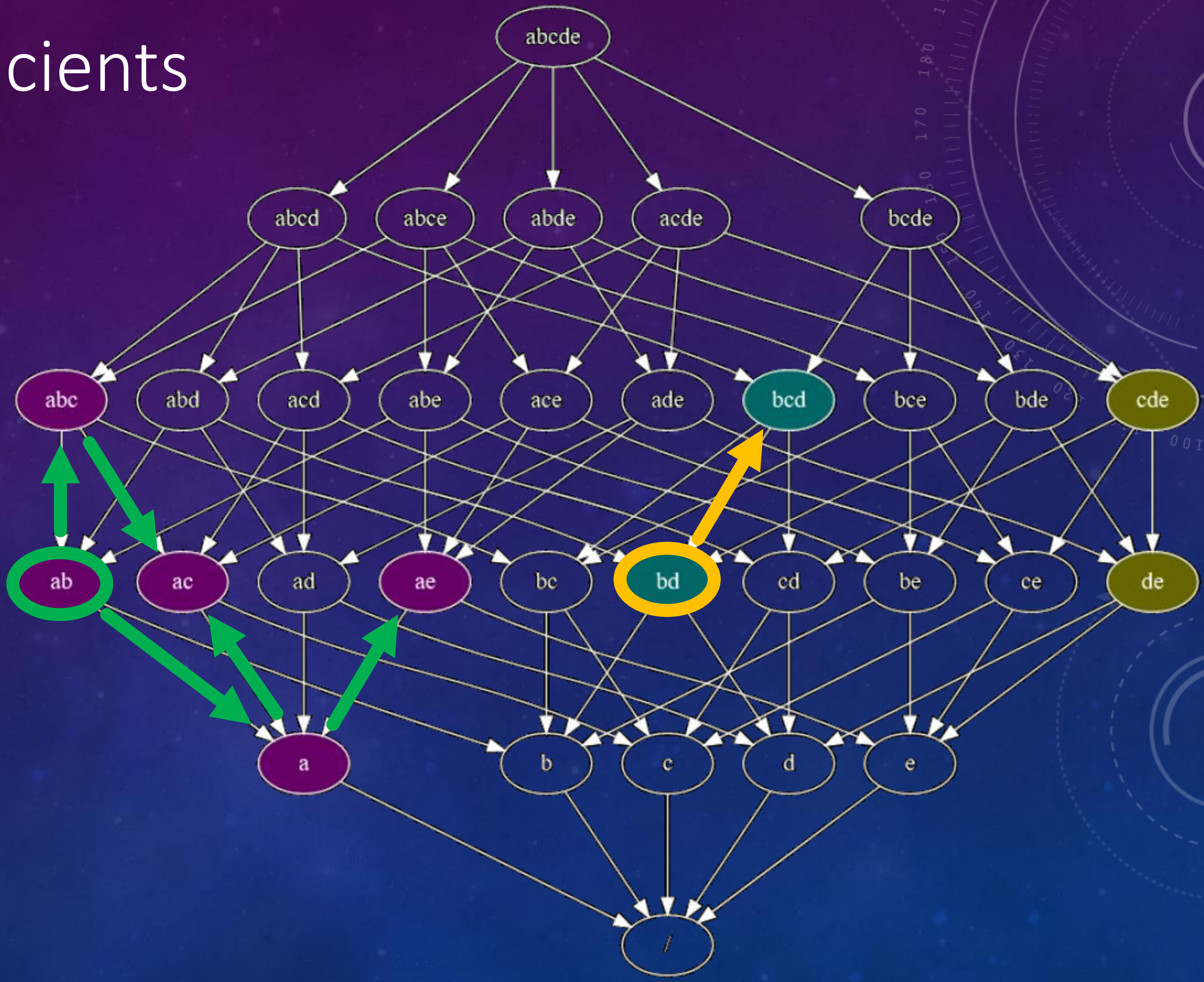
$$D(n+2) = \sum_{\substack{\alpha,\beta \in A_n \\ \alpha \leq \beta}} |[\bot, \alpha]| P_{n,2,\alpha,\beta} |[\beta, \top]|$$

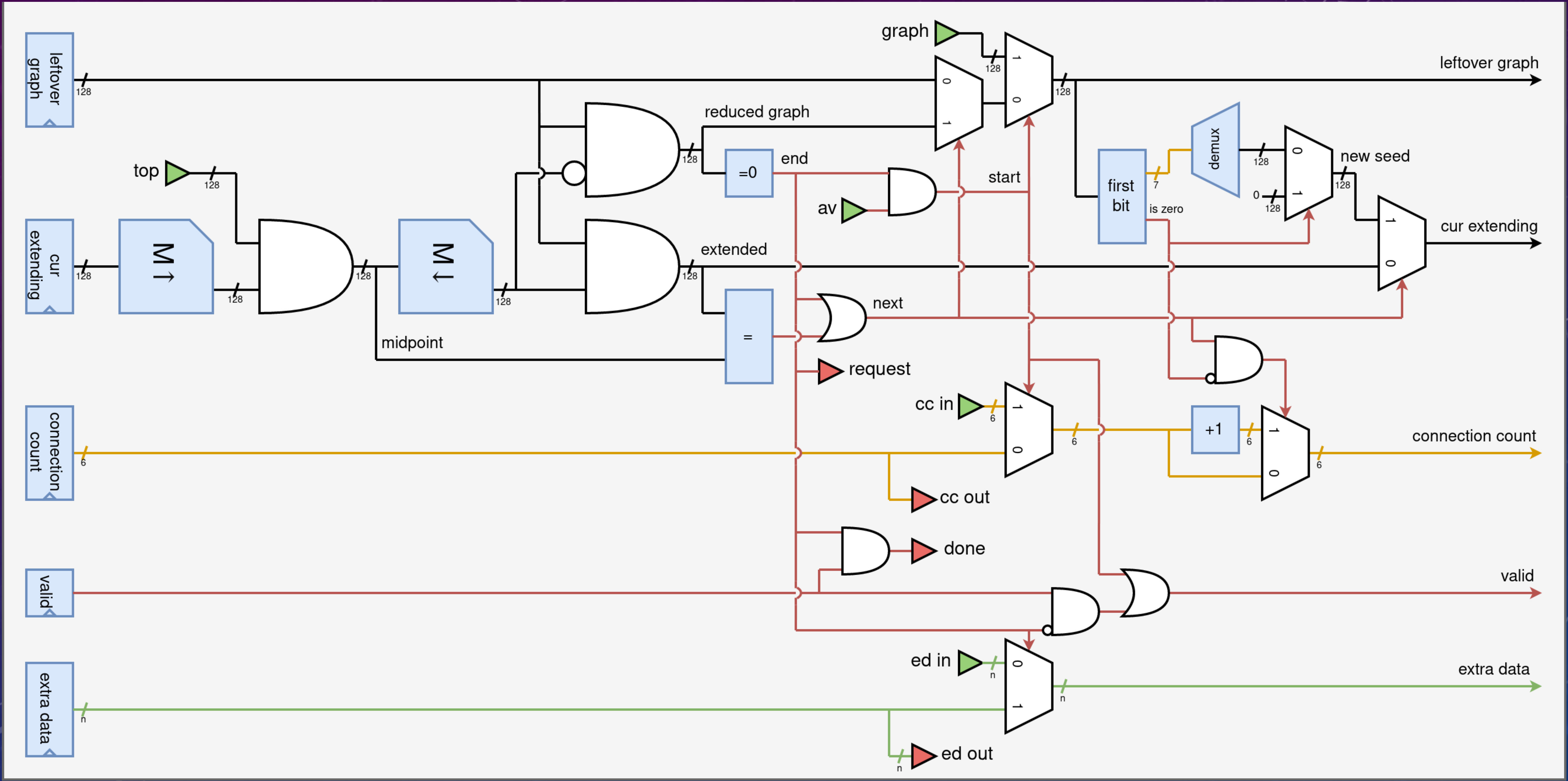
$1{,}148 * 10^{19}$ for D(9)

Computing P-Coëfficients

- Boolean Operations
- Fixed problem size
- Very branchy

- Bad fit for CPU
- Can't use SIMD
- Very bad fit for GPU
- Excellent for FPGA!




leftover graph
cur extending
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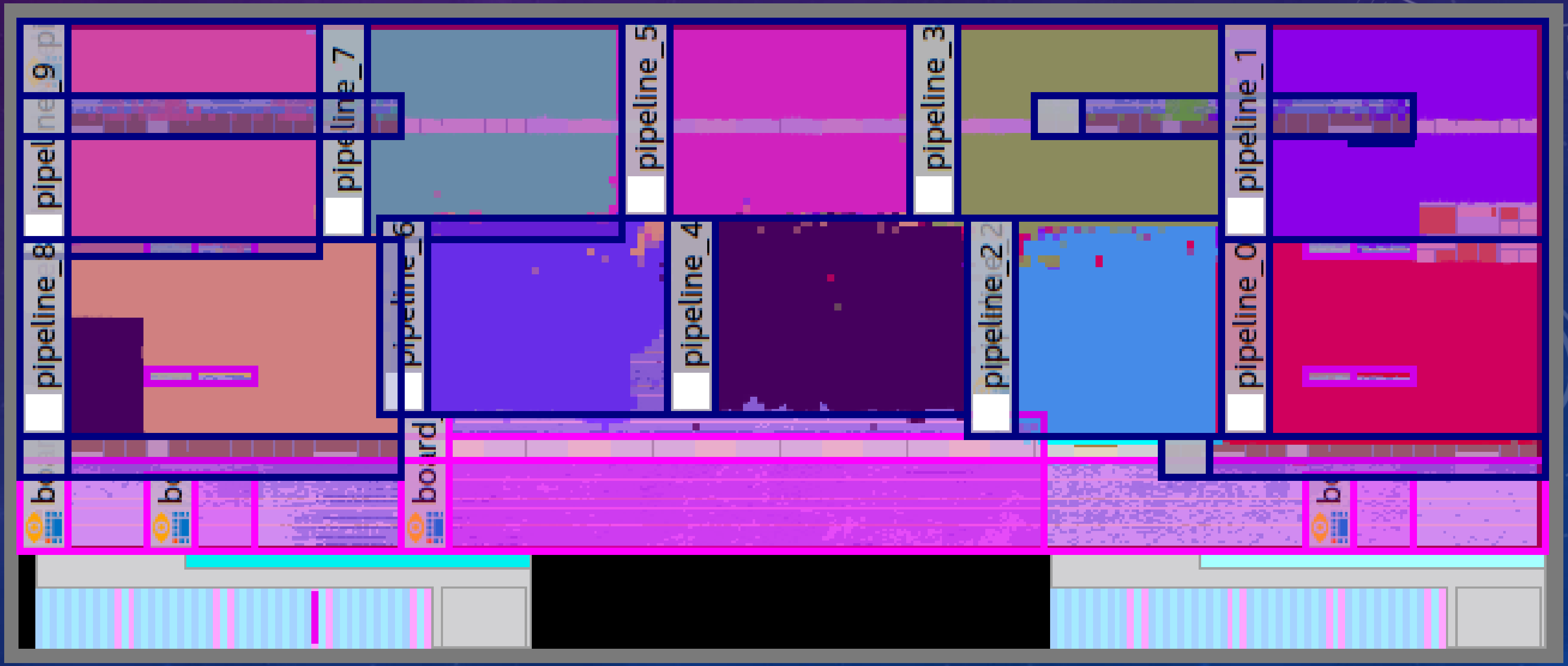
7
6
n
0

FPGA Implementation

- 300 Count-Connected-Cores
- 450MHz
- ECC



Total project runtime

- FPGA 500x faster than AMD Milan CPU.
  - (usually ~40-60x)
- 20’000 70min jobs
- 6 months on Noctua 2

hirtum.com/dedekind